DETERMINANTS (part 4)

AREA OF A TRIANGLE

Area of a Triangle

In earlier classes, we have studied that the area of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3), is given by the expression $\frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$. Now this expression can be written in the form of a determinant as

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \qquad \dots (1)$$

REMARKS

1) Since area is a positive quantity, we always take the absolute value of the determinant. $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

2) If area is given, use both positive and negative values of the determinant for calculation.

   Eg: Find the value of x if the area of Δ is 35 square cms with vertices (x,4) , (2, -6) and (5, 4).

   $\pm 35 = \frac{1}{2}\begin{vmatrix} x & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}$ then solve for x.

3) The area of the triangle formed by three collinear points is zero.

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Q.Find the area of the triangle with vertices (3,8) , (-4,2) and (5,-1)

(i) Given (3, 8), (-4, 2) and (5, -1) are the vertices of the triangle.

We know that, if vertices of a triangle are (x_1, y_1), (x_2, y_2) and (x_3, y_3), then the area of the triangle is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Now, substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & -1 & 1 \end{vmatrix}$$

Expanding along R_1

$$= \frac{1}{2}\left[3\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} - 8\begin{vmatrix} -4 & 1 \\ 5 & 1 \end{vmatrix} + 1\begin{vmatrix} -4 & 2 \\ 5 & -1 \end{vmatrix}\right]$$

$$= \frac{1}{2}[3(3) - 8(-9) + 1(-6)]$$

$$= \frac{1}{2}[9 + 72 - 6]$$

$$= \frac{75}{2}$$ Square units

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Q .Using the determinants show that the following points are collinear:
(i) (5, 5), (-5, 1) and (10, 7)

Solution:

(i) Given (5, 5), (-5, 1) and (10, 7)

We have the condition that three points to be collinear, the area of the triangle formed by these points will be zero. Now, we know that, vertices of a triangle are (x_1, y_1), (x_2, y_2) and (x_3, y_3), then the area of the triangle is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Now, substituting given value in above formula

$$\Delta = \frac{1}{2}\begin{vmatrix} 5 & 5 & 1 \\ -5 & 1 & 1 \\ 10 & 7 & 1 \end{vmatrix} = 0$$

$$\frac{1}{2}\begin{vmatrix} 5 & 5 & 1 \\ -5 & 1 & 1 \\ 10 & 7 & 1 \end{vmatrix}$$

Expanding along R_1

$$= \frac{1}{2}\left[5\begin{vmatrix} 1 & 1 \\ 7 & 1 \end{vmatrix} - 5\begin{vmatrix} -5 & 1 \\ 10 & 1 \end{vmatrix} + 1\begin{vmatrix} -5 & 1 \\ 10 & 7 \end{vmatrix}\right]$$

$$= \frac{1}{2}[5(-6) - 5(-15) + 1(-45)]$$

$$= \frac{1}{2}[-35 + 75 - 45]$$

$= 0$

Area of triangle is zero . Hence points are collinear

Q . If the points (a,0), (0,b) and (1,1) are collinear,prove that a + b = ab

. If the points (a, 0), (0, b) and (1, 1) are collinear, prove that a + b = ab

Solution:

Given (a, 0), (0, b) and (1, 1) are collinear

We have the condition that three points to be collinear, the area of the triangle formed by these points will be zero. Now, we know that, vertices of a triangle are (x_1, y_1), (x_2, y_2) and (x_3, y_3), then the area of the triangle is given by,

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Thus

$$\frac{1}{2}\begin{vmatrix} a & 0 & 1 \\ 0 & b & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

Expanding along R_1

$$\Rightarrow 0 = \frac{1}{2}\left[a\begin{vmatrix} b & 1 \\ 1 & 1 \end{vmatrix} - 0\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 0 & b \\ 1 & 1 \end{vmatrix}\right]$$

$$\Rightarrow \frac{1}{2}\left[a(b-1) - 0(-1) + 1(-b)\right] = 0$$

$\Rightarrow$

$$\frac{1}{2}[ab - a - b] = 0$$

$\Rightarrow$ a + b = ab

Hence Proved

Find the value of x if the area of Δ is 35 square cms with vertices (x, 4), (2, -6) and (5, 4).

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Now, by substituting given value in above formula

$$\Rightarrow \pm 35 = \frac{1}{2}\left|\begin{vmatrix} x & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}\right|$$

$$\Rightarrow \pm 70 = \begin{vmatrix} x & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}$$

Expanding along R_1

$$\Rightarrow \left[x\begin{vmatrix} -6 & 1 \\ 4 & 1 \end{vmatrix} - 4\begin{vmatrix} 2 & 1 \\ 5 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & -6 \\ 5 & 4 \end{vmatrix}\right] = \pm 70$$

$\Rightarrow [x(-10) - 4(-3) + 1(8 - 30)] = \pm 70$

$\Rightarrow [-10x + 12 + 38] = \pm 70$

$\Rightarrow \pm 70 = -10x + 50$

Taking positive sign, we get

$\Rightarrow +70 = -10x + 50$

$\Rightarrow 10x = -20$

$\Rightarrow x = -2$

Taking –negative sign, we get

$\Rightarrow -70 = -10x + 50$

$\Rightarrow 10x = 120$

$\Rightarrow x = 12$

Thus $x = -2, 12$

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Find the equation of the line joining(1,2) and (3,6)using determinants

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Here the vertices are (1,2), (3,6) and Let the 3rd vetex is P (x,y)

Since these points are collinear, Δ = 0

$$0 = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 3 & 6 & 1 \\ x & y & 1 \end{vmatrix}$$

0 = 1 (6-y) -2(3-x) + 1 (3y-6x)

2x - y = 0 is the equation of the line.

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HOME WORK

Ex: 4.3

Q . 2 , 3, 4(ii)

DETRMINANTS(PART 5)

MINORS,CO FACTORS,ADJOINT & INVERSE OF A MATRIX

MINORS

MINORS

Minor of an element a_{ij} of a determinant is the determinant obtained by deleting its i^{th} row and j^{th} column in which element a_{ij} lies. Minor of an element a_{ij} is denoted by M_{ij}.

Eg: Minor of a_{11} (M_{11}) $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = |4| = 4$

Minor of a_{21} (M_{21}) $= \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = |2| = 2$

Minor of a_{12} (M_{12}) $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = |3| = 3$

Minor of a_{22} (M_{22}) $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = |1| = 1$

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MINORS

Minor of an element a_{ij} of a determinant is the determinant obtained by deleting its i^{th} row and j^{th} column in which element a_{ij} lies. Minor of an element a_{ij} is denoted by M_{ij}.

Eg: Minor of a_{11} (M_{11}) $\begin{vmatrix} 2 & 1 & 3 \\ 4 & 0 & 5 \\ 6 & 7 & 9 \end{vmatrix} = (0\times9)-(5\times7) = -35$

Minor of a_{23} (M_{23}) $\begin{vmatrix} 2 & 1 & 3 \\ 4 & 0 & 5 \\ 6 & 7 & 9 \end{vmatrix} = (2\times7) - (1\times6) = 8$

COFACTORS

COFACTORS

Cofactor of an element a_{ij}, denoted by A_{ij} is defined by $A_{ij} = (-1)^{i+j} M_{ij}$, where M_{ij} is minor of a_{ij}

Cofactor of a_{11} (A_{11}) $\begin{vmatrix} 2 & 1 & 3 \\ 4 & 0 & 5 \\ 6 & 7 & 9 \end{vmatrix} = (-1)^{1+1} M_{11}$

$= (-1)^{1+1} (0\times9)-(5\times7) = -35$

Cofactor of a_{23} (A_{23}) $\begin{vmatrix} 2 & 1 & 3 \\ 4 & 0 & 5 \\ 6 & 7 & 9 \end{vmatrix} = (-1)^{2+3} M_{11}$

$= - [(2\times7) - (1\times6)] = - 8$

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Eg:

Q) Find the minors and cofactors of the elements of the determinant $\begin{vmatrix} 1 & -2 \\ 4 & 3 \end{vmatrix}$

Here $a_{11} = 1$. So M_{11} = Minor of $a_{11} = 3$

M_{12} = Minor of the element $a_{12} = 4$

M_{21} = Minor of the element $a_{21} = -2$

M_{22} = Minor of the element $a_{22} = 1$

Now, cofactor of a_{ij} is A_{ij}. So

$A_{11} = (-1)^{1+1}\ M_{11} = (-1)^2\ (3) = 3$

$A_{12} = (-1)^{1+2}\ M_{12} = (-1)^3\ (4) = -4$

$A_{21} = (-1)^{2+1}\ M_{21} = (-1)^3\ (-2) = 2$

$A_{22} = (-1)^{2+2}\ M_{22} = (-1)^4\ (1) = 1$

Eg:

Q) Find the minors and cofactors of an elements of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

We have $M_{11} = \begin{vmatrix} 0 & 4 \\ 5 & -7 \end{vmatrix} = 0 - 20 = -20;\quad A_{11} = (-1)^{1+1}(-20) = -20$

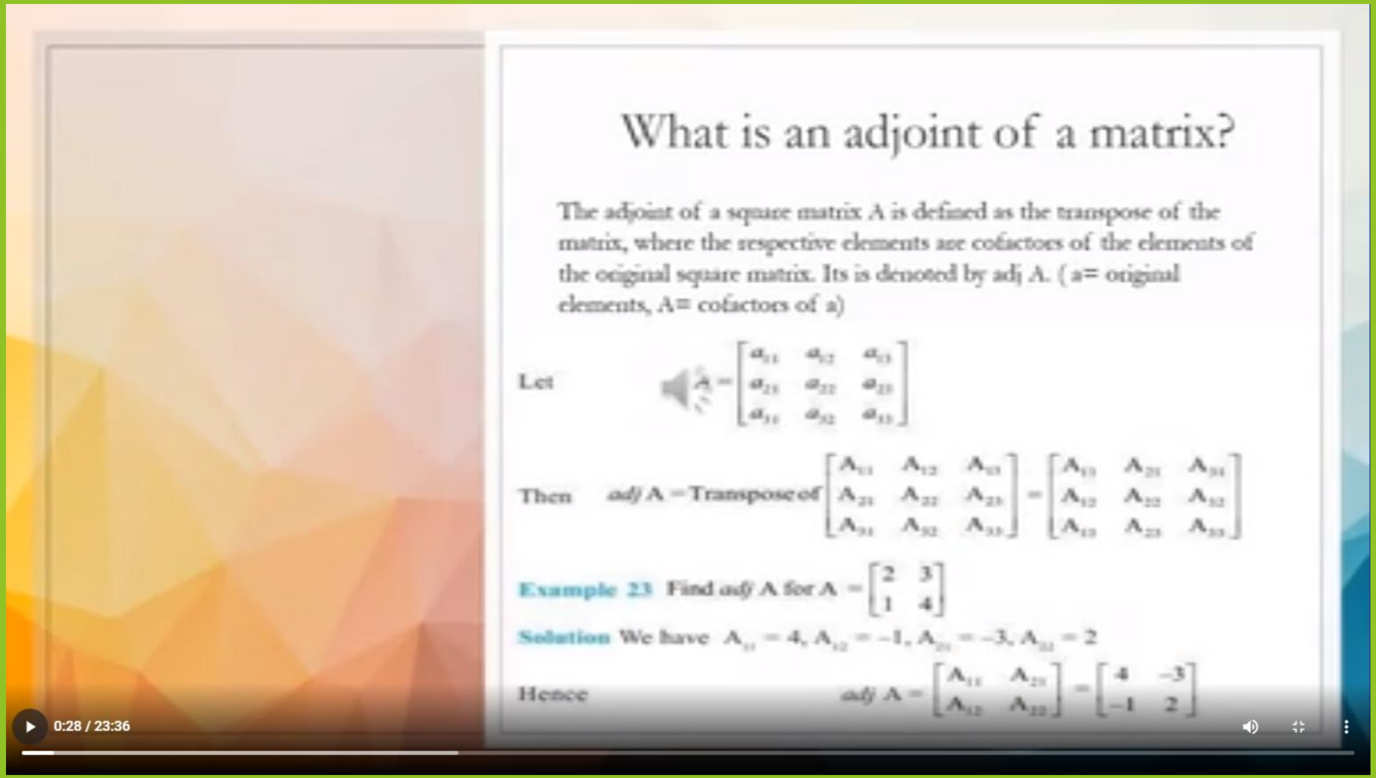
$M_{12} = \begin{vmatrix} 6 & 4 \\ 1 & -7 \end{vmatrix} = -42 - 4 = -46;\quad A_{12} = (-1)^{1+2}(-46) = 46$

$M_{13} = \begin{vmatrix} 6 & 0 \\ 1 & 5 \end{vmatrix} = 30 - 0 = 30;\quad A_{13} = (-1)^{1+3}(30) = 30$

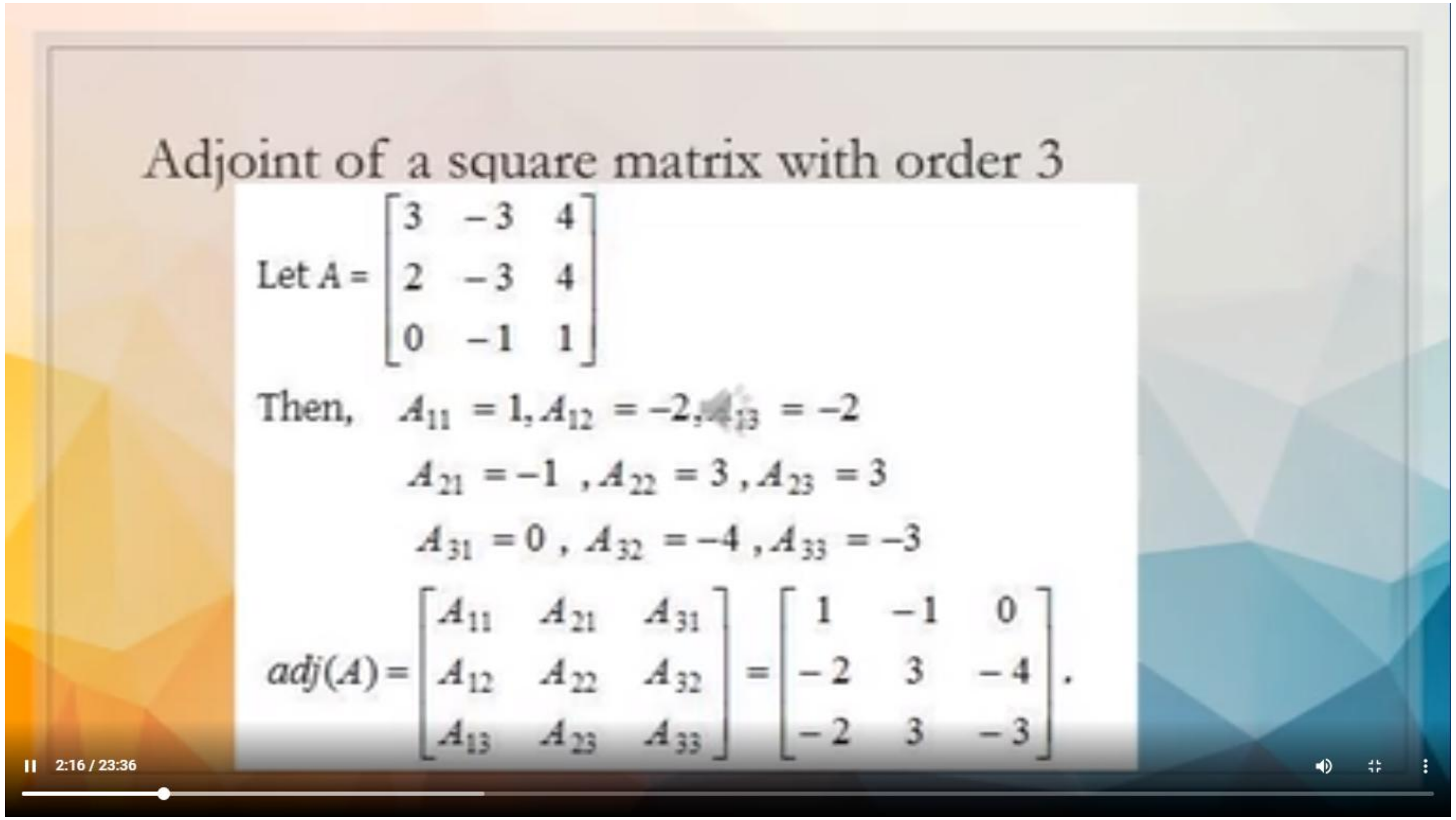
$M_{21} = \begin{vmatrix} -3 & 5 \\ 5 & -7 \end{vmatrix} = 21 - 25 = -4;\quad A_{21} = (-1)^{2+1}(-4) = 4$

$M_{22} = \begin{vmatrix} 2 & 5 \\ 1 & -7 \end{vmatrix} = -14 - 5 = -19;\quad A_{22} = (-1)^{2+2}(-19) = -19$

ADJOINT


What is an adjoint of a matrix?
The adjoint of a square matrix A is defined as the transpose of the matrix, where the respective elements are cofactors of the elements of the original square matrix. Its is denoted by adj A. (a= original elements, A= cofactors of a)
Let
Then adj A = Transpose of
Example 23 Find adj A for A =
Solution We have $A_{11} = 4, A_{12} = -1, A_{21} = -3, A_{22} = 2$
Hence adj A =
0:28 / 23:36

Eg: to find Adjoint of a matrix



Adjoint of a square matrix with order 3

Let $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

Then, $A_{11} = 1, A_{12} = -2, A_{13} = -2$

$A_{21} = -1, A_{22} = 3, A_{23} = 3$

$A_{31} = 0, A_{32} = -4, A_{33} = -3$

$$adj(A) = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}.$$

Theorem

$A(adjA) = |A|I = (adjA)A$

$\rightarrow \frac{1}{|A|}A(adjA) = I$

$\rightarrow A\,[\frac{1}{|A|}(adjA)] = I$ [If AB = I =BA,Then B=A^{-1}]

$\rightarrow A\,(A^{-1}) = I$

$\therefore A^{-1} = \frac{1}{|A|}(adjA)$

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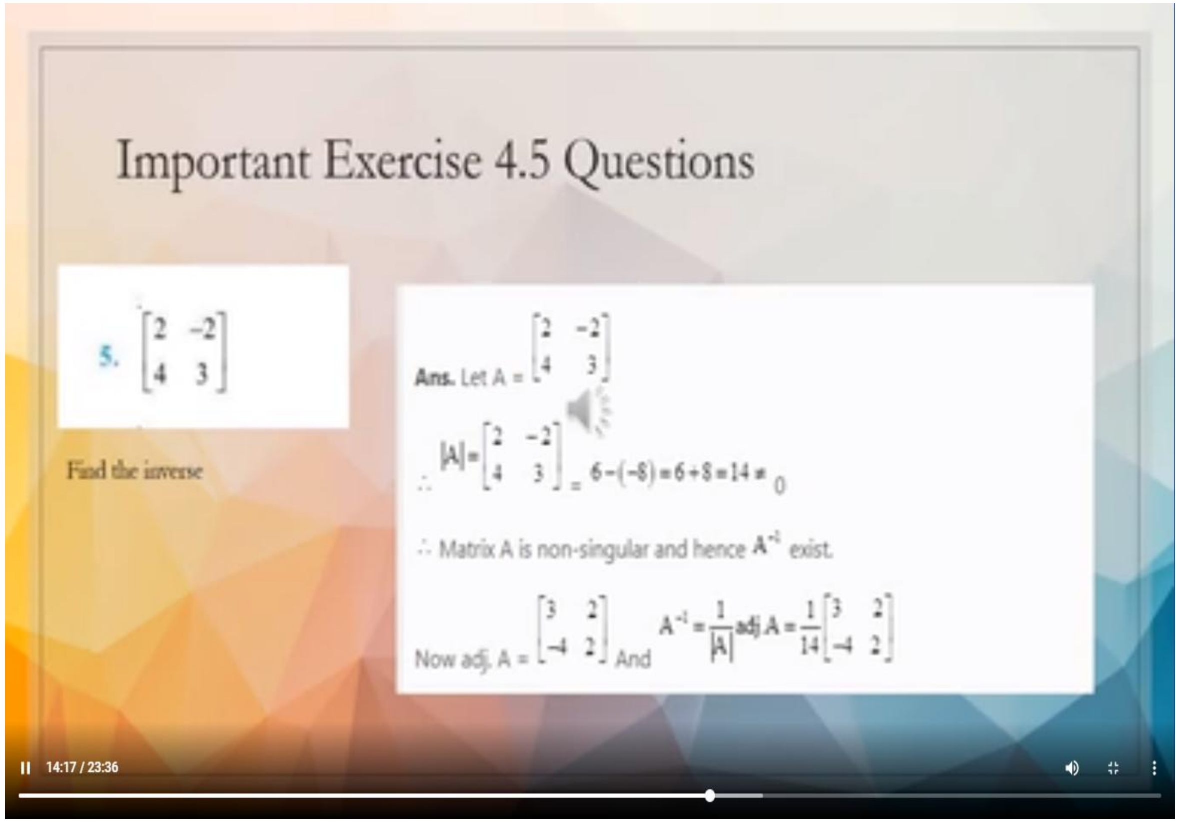
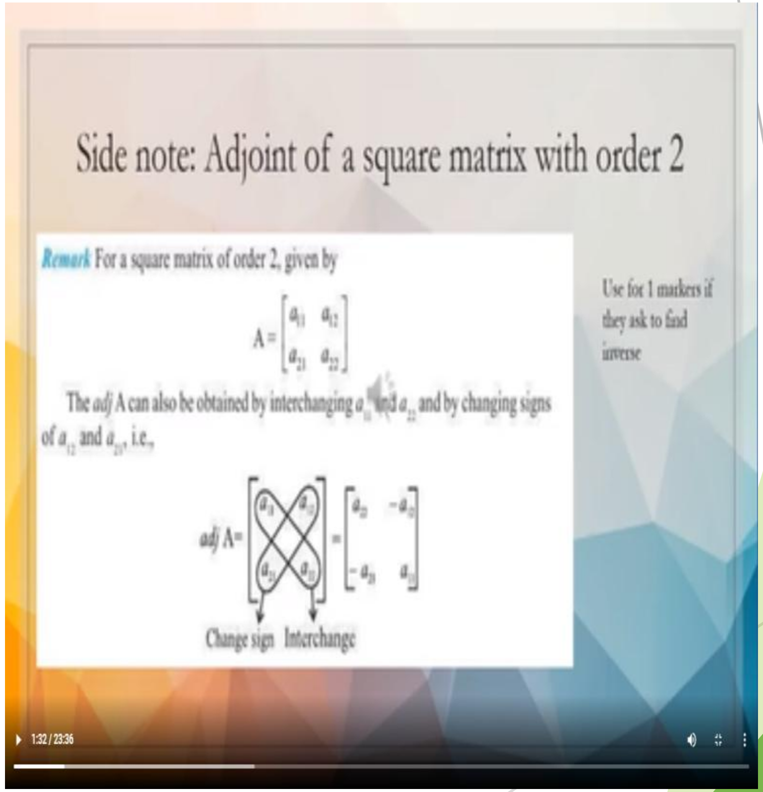
FIND THE INVERSE OF A MATRIX (2x2)





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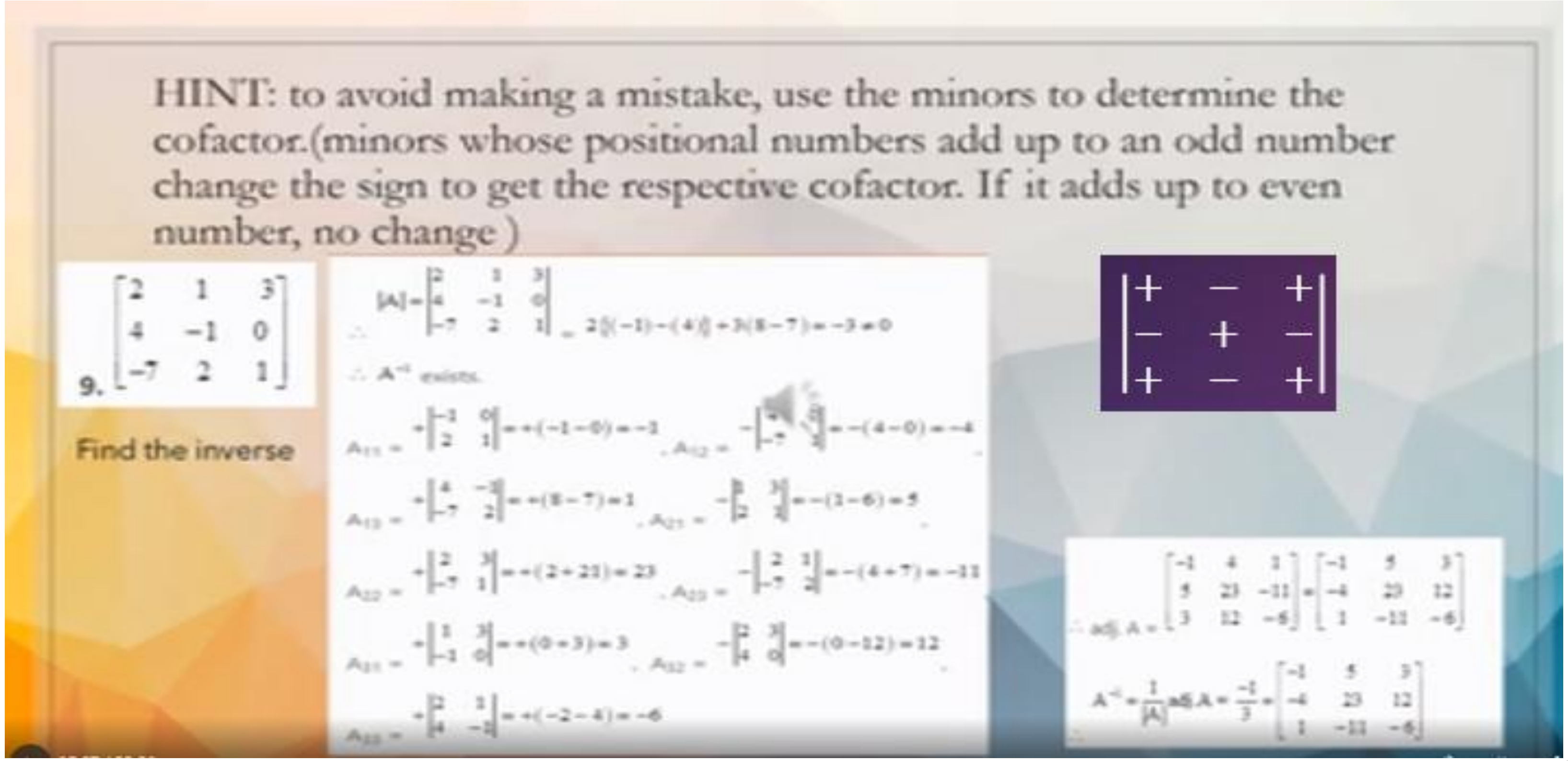
FIND THE INVERSE OF A MATRIX (3x3)



HINT: to avoid making a mistake, use the minors to determine the cofactor.(minors whose positional numbers add up to an odd number change the sign to get the respective cofactor. If it adds up to even number, no change)

9. $\begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$

Find the inverse

$|A| = \begin{vmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{vmatrix} = 2[(-1)-(4)] + 3(8-7) = -3 \neq 0$

$\therefore A^{-1}$ exists.

$A_{11} = +\begin{vmatrix} -1 & 0 \\ 2 & 1 \end{vmatrix} = +(-1-0) = -1$, $A_{12} = -\begin{vmatrix} 4 & 0 \\ -7 & 1 \end{vmatrix} = -(4-0) = -4$

$A_{13} = +\begin{vmatrix} 4 & -1 \\ -7 & 2 \end{vmatrix} = +(8-7) = 1$, $A_{21} = -\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = -(1-6) = 5$

$A_{22} = +\begin{vmatrix} 2 & 3 \\ -7 & 1 \end{vmatrix} = +(2+21) = 23$, $A_{23} = -\begin{vmatrix} 2 & 1 \\ -7 & 2 \end{vmatrix} = -(4+7) = -11$

$A_{31} = +\begin{vmatrix} 1 & 3 \\ -1 & 0 \end{vmatrix} = +(0+3) = 3$, $A_{32} = -\begin{vmatrix} 2 & 3 \\ 4 & 0 \end{vmatrix} = -(0-12) = 12$

$A_{33} = +\begin{vmatrix} 2 & 1 \\ 4 & -1 \end{vmatrix} = +(-2-4) = -6$

$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$

$\therefore \text{adj. } A = \begin{bmatrix} -1 & 4 & 1 \\ 5 & 23 & -11 \\ 3 & 12 & -6 \end{bmatrix}' = \begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj} A = \frac{-1}{3} \begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$

IMPORTANT RESULTS AND THEOREMS

1) $A(adjA) = |A|I = (adjA)A$

2) $|adjA| = |A|^{n-1}$

3) $|A\ adjA| = |A|^{n}$

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HOME WORK

EX 4.5

Q .8,10 ,13,14,15

DETERMINANTS

Part 6

TESSY ROY VARGHESE
INDIAN SCHOOL MUSCAT

TOPICS

- APPLICATION OF DETERMINANTS AND MATRICES IN SOLVING SYSTEM OF LINEAR EQUATIONS IN TWO OR THREE VARIABLES
- APPLICATION OF DETERMINANTS AND MATRICES IN CHECKING THE CONSISTENCY OF THE SYSTEM OF LINEAR EQUATIONS
- NOTE: WE RESTRICT OURSELVES TO THE SYSTEM OF LINEAR EQUATIONS

  HAVING UNIQUE SOLUTION ONLY

How do we apply?

In the previous chapter, we have studied about matrices and algebra of matrices. We have also learnt that a system of algebraic equations can be expressed in the form of matrices. This means, a system of linear equations like

$$a_1 x + b_1 y = c_1$$
$$a_2 x + b_2 y = c_2$$

can be represented as $\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$.

Let $A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$

Then, $AX = B$

$\therefore X = A^{-1}B$

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CONDITIONS FOR CONSISTENT & INCONSISTENT EQUATIONS

- X = A^{-1}B; A^{-1}= $\frac{1}{|A|}$ (adjA)

- Case1: If $|A|$ = any constant other than zero, A^{-1} exists.

  The system of equations is said to be consistent. And it has one or more solutions

- Case 2: If $|A|$ = 0 (A is a singular matrix);

  Find (adjA) B and if (adjA) B = 0 ,then the system may be either consistent or inconsistent according as the system has either infinitely many solutions or no solution.

- Case 3 : : If $|A|$ = 0 and (adjA) B ≠ 0 ,then solution does not exist and the system of equations is inconsistent

Eg:CHECKING CONSISTENCY

Question

$2x - y = 5$

$x + y = 4$

Answer

The given system of equations: $\begin{matrix} 2x - y = 5 \\ x + y = 4 \end{matrix}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$|A| = 2 + 1 = 3 \neq 0 \quad \Rightarrow A$ is non-singular and so A^{-1} exists.

Hence, the system of equations are consistent.

Question

$x + 3y = 5$

$2x + 6y = 8$

Answer

The given system of equations: $\begin{matrix} x + 3y = 5 \\ 2x + 6y = 8 \end{matrix}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

$|A| = 6 - 6 = 0 \quad \Rightarrow A$ is a singular matrix and so A^{-1} does not exists. Now,

$$adj\ A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$$

$$(adj\ A)B = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 30 - 24 \\ -10 + 8 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix} \neq O$$

So, there is no solutions of the given system of equations.

Hence, the system of equations are inconsistent.

SOLVE, IF CONSISTENT

Question

$2x - y = -2$

$3x + 4y = 3$

Answer

The given system of equations: $\begin{matrix} 2x - y = -2 \\ 3x + 4y = 3 \end{matrix}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$|A| = 8 + 3 = 11 \neq 0 \quad \Rightarrow$ A is non-singular and so A^{-1} exists. Now,

Hence, the system of equations are consistent.

Now, $A_{11} = 4 \quad A_{12} = -3 \quad A_{21} = 1 \quad A_{22} = 2$

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{|A|}\begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$$

$$X = A^{-1}B \quad \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -8 + 3 \\ 6 + 6 \end{bmatrix} \quad \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{5}{11} \\ \frac{12}{11} \end{bmatrix} \quad \Rightarrow x = -\frac{5}{11}, \quad y = \frac{12}{11}$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

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Eg:2

Question

$$x - y + z = 4$$
$$2x + y - 3z = 0$$
$$x + y + z = 2$$

Answer

The given system of equations: $\begin{aligned} x - y + z &= 4 \\ 2x + y - 3z &= 0 \\ x + y + z &= 2 \end{aligned}$

This system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$|A| = 1(1+3) + 1(2+3) + 1(2-1) = 4 + 5 + 1 = 10 \neq 0$

$\Rightarrow$ A is non-singular and so A^{-1} exists. Now,

$A_{11} = 4$ $\quad A_{12} = -5$ $\quad A_{13} = 1$

$A_{21} = 2$ $\quad A_{22} = 0$ $\quad A_{23} = -2$

$A_{31} = 2$ $\quad A_{32} = 5$ $\quad A_{33} = 3$

$$\text{adj A} = \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}^T = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$X = A^{-1}B \quad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 16 + 0 + 4 \\ -20 + 0 + 10 \\ 4 + 0 + 6 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \Rightarrow x = 2, y = -1, z = 1$$

USING INVERSE OF A , SOLVE:

Question

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations

$$\begin{aligned} 2x - 3y + 5z &= 11 \\ 3x + 2y - 4z &= -5 \\ x + y - 2z &= -3 \end{aligned}$$

Answer

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$|A| = 2(-4 + 4) + 3(-6 + 4) + 5(3 - 2) = 0 - 6 + 5 = -1 \neq 0$

$\Rightarrow$ A is non-singular and so A^{-1} exists. Now,

$A_{11} = 0$ $\quad A_{12} = 2$ $\quad A_{13} = 1$

$A_{21} = -1$ $\quad A_{22} = -9$ $\quad A_{23} = -5$

$A_{31} = 2$ $\quad A_{32} = 23$ $\quad A_{33} = 13$

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

The given system of equations: $\begin{aligned} 2x - 3y + 5z &= 11 \\ 3x + 2y - 4z &= -5 \\ x + y - 2z &= -3 \end{aligned}$

This system of equations can be written as $AX = B$, where

$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

$$X = A^{-1}B \quad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 - 5 + 6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\Rightarrow x = 1, y = 2, z = 3$

Ex 33. PROBLEM

Use product $\begin{bmatrix}1 & -1 & 2\\0 & 2 & -3\\3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\9 & 2 & -3\\6 & 1 & -2\end{bmatrix}$ to solve the system of equations

$x - y + 2z = 1$

$2y - 3z = 1$

$3x - 2y + 4z = 2$

Consider the product

$$\begin{bmatrix}1 & -1 & 2\\0 & 2 & -3\\3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\9 & 2 & -3\\6 & 1 & -2\end{bmatrix} = \begin{bmatrix}-2-9+12 & 0-2+2 & 1+3-4\\0+18-18 & 0+4-3 & 0-6+6\\-6-18+24 & 0-4+4 & 3+6-8\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

We know that

$AA^{-1} = I$

So $\begin{bmatrix}-2 & 0 & 1\\0 & 2 & -3\\6 & 1 & -2\end{bmatrix}$ is inverse of $\begin{bmatrix}1 & -1 & 2\\0 & 2 & -3\\3 & -2 & 4\end{bmatrix}$

i.e. $\begin{bmatrix}1 & -1 & 2\\0 & 2 & -3\\3 & -2 & 4\end{bmatrix}^{-1} = \begin{bmatrix}-2 & 0 & 1\\9 & 2 & -3\\6 & 1 & -2\end{bmatrix}$

Writing the equation as AX = B

$$\begin{bmatrix}1 & -1 & 2\\0 & 2 & -3\\3 & -2 & 4\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\1\\2\end{bmatrix}$$

Here A = $\begin{bmatrix}1 & -1 & 2\\0 & 2 & -3\\3 & -2 & 4\end{bmatrix}$

So, $A^{-1} = \begin{bmatrix}1 & -1 & 2\\0 & 2 & -3\\3 & -2 & 4\end{bmatrix}^{-1} = \begin{bmatrix}-2 & 0 & 1\\9 & 2 & -3\\6 & 1 & -2\end{bmatrix}$

$$\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}-2 & 0 & 1\\9 & 2 & -3\\6 & 1 & -2\end{bmatrix}\begin{bmatrix}1\\1\\2\end{bmatrix} =$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}0\\5\\3\end{bmatrix}$$

HOME WORK

EX : 4.6

Q: 11,12,13,14

THANK YOU

DETERMINANTS –PART 7

MIS .EX

Q1 Solve:

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4$$
$$\frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1$$
$$\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$$

This system of equations can be written as $AX = B$, where

$$A=\begin{bmatrix}2 & 3 & 10\\ 4 & -6 & 5\\ 6 & 9 & -20\end{bmatrix}, X=\begin{bmatrix}1/x\\1/y\\1/z\end{bmatrix} \text{ and } B=\begin{bmatrix}4\\1\\2\end{bmatrix}$$

$$|A| = 2(120-45)-3(-80-30)+10(36+36) = 150+330+720 = 1200 \neq 0$$

$A_{11}=75$ $A_{12}=110$ $A_{13}=72$

$A_{21}=150$ $A_{22}=-100$ $A_{23}=0$

$A_{31}=75$ $A_{32}=30$ $A_{33}=-24$

$$A^{-1}=\frac{1}{|A|}adj\,A=\frac{1}{1200}\begin{bmatrix}75 & 150 & 75\\110 & -100 & 30\\72 & 0 & -24\end{bmatrix}$$

$X = A^{-1}B$

$$\Rightarrow \begin{bmatrix}1/x\\1/y\\1/z\end{bmatrix}=\frac{1}{1200}\begin{bmatrix}75 & 150 & 75\\110 & -100 & 30\\72 & 0 & -24\end{bmatrix}\begin{bmatrix}4\\1\\2\end{bmatrix}$$

$$\begin{bmatrix}\frac{1}{x}\\\frac{1}{y}\\\frac{1}{z}\end{bmatrix}=\frac{1}{1200}\begin{bmatrix}600\\400\\240\end{bmatrix}=\begin{bmatrix}\frac{1}{2}\\\frac{1}{3}\\\frac{1}{5}\end{bmatrix}$$

$$\Rightarrow \frac{1}{x}=\frac{1}{2},\quad \frac{1}{y}=\frac{1}{3},\quad \frac{1}{z}=\frac{1}{5}$$

x = 2,y=3, z = 5

I S M

Q 2

Q2 The cost of 4 kg onion, 3 kg wheat and 2 kg rice is ₹60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is ₹90. The cost of 6 kg onion 2 kg wheat and 3 kg rice is ₹70. Find cost of each item per kg by matrix method.

Let x, y and z be the prices of onion, wheat and rice per kg

$4x+3y+2z=60$

$2x+4y+6z=90$

$6x+2y+3z=70$

Matrix form of given equations is AX = B

$$\Rightarrow \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$\Rightarrow X = A^{-1}B = \frac{1}{|A|}(\text{adj. } A)B$$

$$|A| = 50$$

$$\text{adj}A = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}\begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

Cost of onion, wheat and rice per Kg are ₹5, ₹8 and ₹8.

I S M

Q 3

Q3

Prove: $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = 1$

$$\Delta = \begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & 2+p \\ 0 & 3 & 7+3p \end{vmatrix} \qquad \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

= (7+3p)-(6+3p) = 1 *Opening along C_1*

Q3

If a, b and c are in AP, then evaluate $\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$

$$\Delta = \begin{vmatrix} 2x+6 & 2x+8 & 2x+2(a+c) \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} \qquad R_1 \to R_1 + R_3$$

$$\Delta = \begin{vmatrix} 2(x+3) & 2(x+4) & 2(x+2b) \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} \qquad 2b = a + c$$

$$R_1 = 2R_2$$

$$\Delta = 0$$

13:29

Q 4

***Q* 4** $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ **show that A^2-5A +7I = O..Hence find A^{-1}.**

$A^2 = \begin{pmatrix} 7 & 12 \\ 4 & 7 \end{pmatrix}$: 5A = $\begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$: 7I = $\begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

A^2-5A +7I = $\begin{pmatrix} 7 & 12 \\ 4 & 7 \end{pmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

A^2-5A +7I = O

A.A – 5A + 7I =0

Post multiplying by A^{-1} ,Since $|A| \neq 0$

A.A(A^{-1}) – 5A (A^{-1}) + 7I(A^{-1}) =0(A^{-1})

A.I – 5I + 7A^{-1} = 0

7A^{-1} = 5I – AI = 5I –A

$$A^{-1} = \frac{1}{7}[5I - A] = \frac{1}{7}\left[\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\right] = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{2}{7} & \frac{-1}{7} \\ \frac{1}{7} & \frac{3}{7} \end{bmatrix}$$

I S M

Q5

5)The sum of three numbers is 6.If we multiply the third number by 2 and add the first number to the result, we get 7. By adding second and third numbers to three times the first number, we get 12.Using matrices ,find numbers.

x + y + z = 6
x + 2z = 7
3x + y + z =12

A_{11}= -2 A_{21}= 0 A_{31}=2
A_{12} = 5 A_{22} = -2 A_{32}=-1
A_{13} = 1 A_{23}=2 A_{33} =-1

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix} \rightarrow A X = B \rightarrow X = A^{-1}B$$

$|A| = 4$

$$A^{-1} = \frac{1}{detA}(\text{ adj A}) = \frac{1}{4}\begin{pmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{pmatrix} \rightarrow X = A^{-1}B = \frac{1}{4}\begin{pmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{pmatrix}\begin{pmatrix} 6 \\ 7 \\ 12 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$$

X = 3, y = 1 , z = 2
There fore the three numbers are 3 ,1 , 2

HOME WORK

MISCELLANEOUS EXCERCISES

Q 11, 13, 15, 18, 19

I
S
M